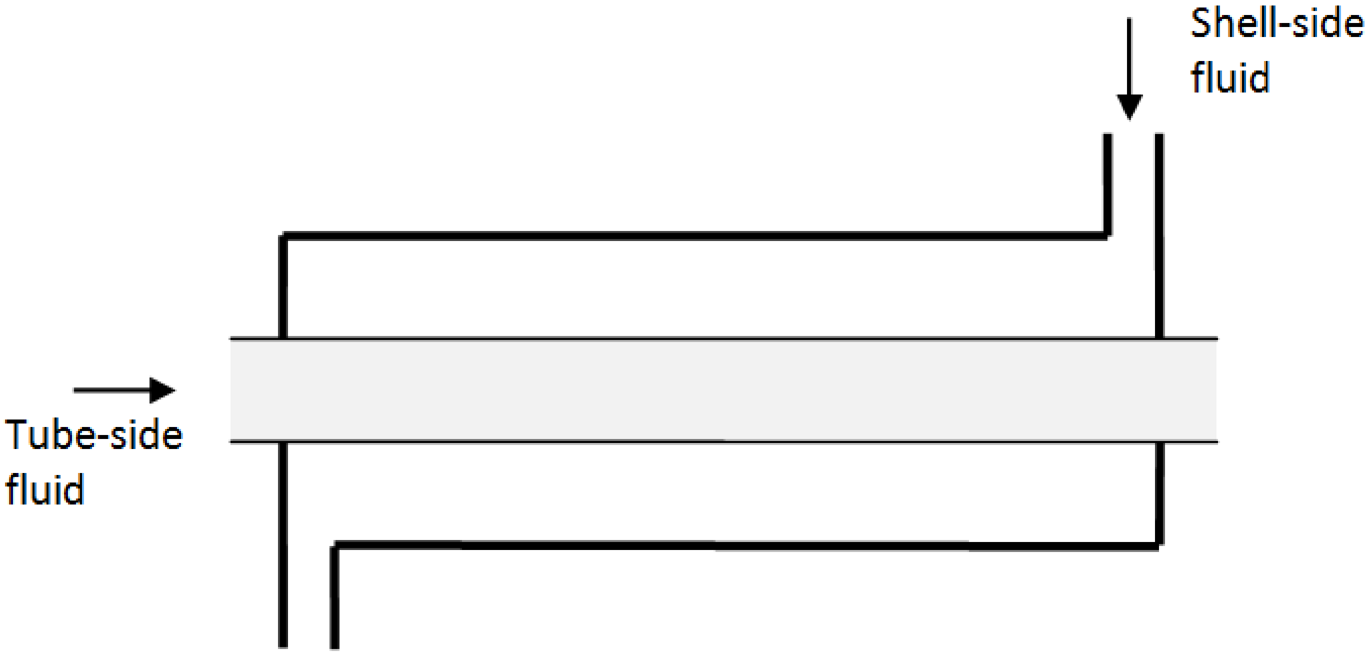
Double-Pipe Heat Exchanger

Introduction

This application models the temperature dynamics of a countercurrent double pipe heat exchanger. Three partial differential equations describe

- heat balances across the tube- and shell-side liquids,
- and a heat balance across the tube-wall (taking into account the heat flow from the shell- and tube-side liquids, and conduction along the length of the tube)

The equations are solved numerically, and the temperature profiles are plotted. The heat exchanger is assumed to be perfectly insulated. Densities, specific heat capacities, heat transfer coefficients, and thermal conductivities are assumed to be constant.



Parameters

Specific heat capacities for the tube-side fluid, shell-side fluid and the tube wall

> Cpt := 4085 : Cps := 4186 : Cpw := 380 :

Densities

> rhot := 800 : rhos := 1200 : rhow := 8000 :

Flowrates

> Ft := 1 : Fs := 1 :

Heat transfer coefficients

> Ut := 40000 : Us := 40000 :

Thermal conductivity of tube-wall

> kw := 109 :

Length of heat exchanger

> L := 1 :

Internal and external diameter of inner tube, and internal diameter of outer tube.

> Di := 0.05 : Do := 0.06 : Dis := 0.1 :

Partial Differential Equations Derived From Energy Balances

Tube wall heat balance

$$> pde1 := \frac{\pi}{4}\left(Do^2 - Di^2\right) Cpw\, rhow \left(\frac{\partial}{\partial t} Tw(x,t)\right) = Ut\,\pi\, Di\,(Tt(x,t) - Tw(x,t)) - Us\,\pi\, Do\,(Tw(x,t) - Ts(x,t)) + kw\,\frac{\pi}{4}\cdot\left(Do^2 - Di^2\right)\left(\frac{\partial^2}{\partial x^2} Tw(x,t)\right) :$$

Tube-side heat balance

$$> pde2 := rhot\,Cpt\,\frac{\pi}{4}\,Di^2\left(\frac{\partial}{\partial t} Tt(x,t)\right) = -Cpt\,Ft\left(\frac{\partial}{\partial x} Tt(x,t)\right) - \pi\,Di\,Ut\,(Tt(x,t) - Tw(x,t)) :$$

Shell-side heat balance

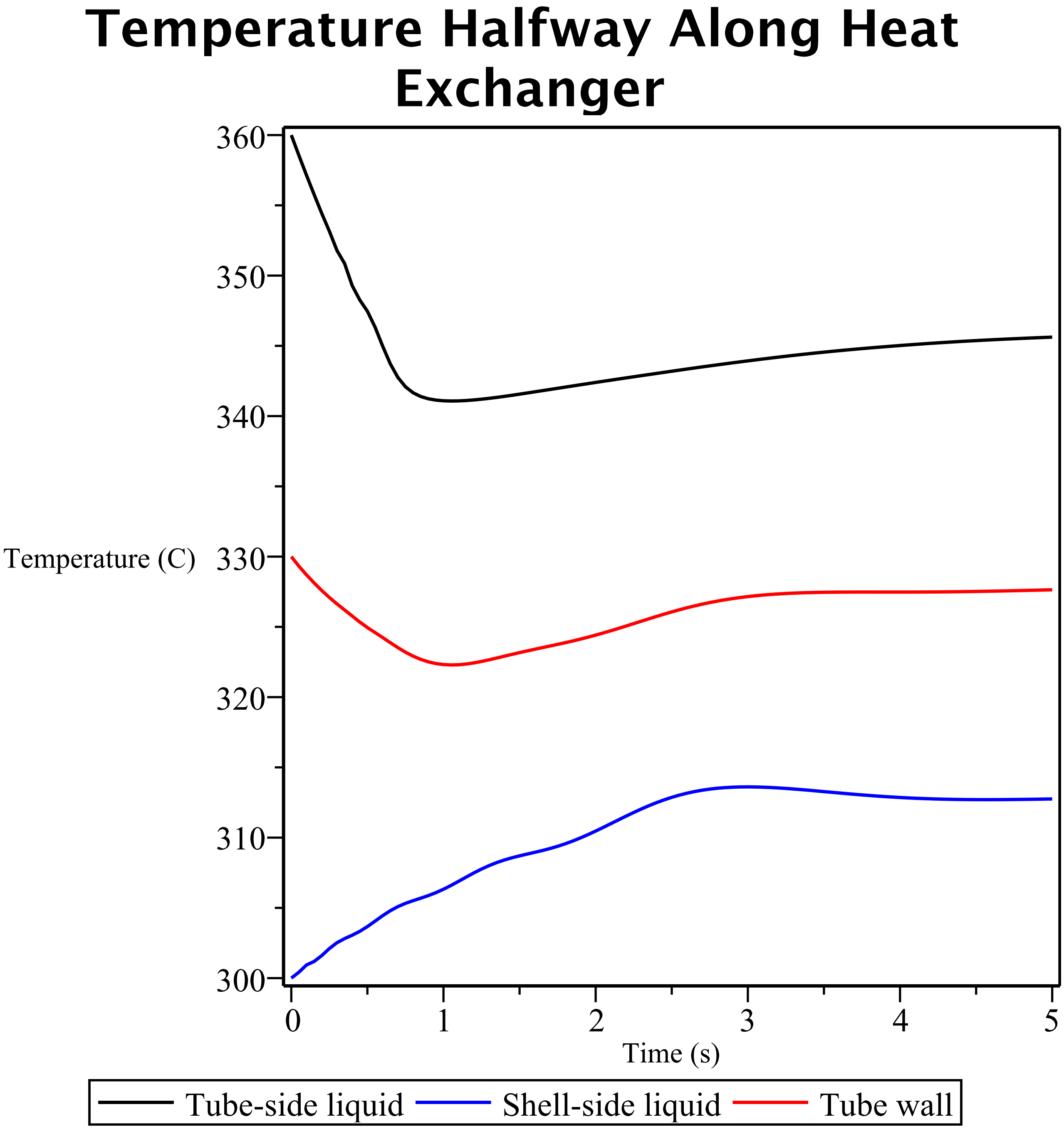
$$> pde3 := rhos\,Cps\,\frac{\pi}{4}\left(Dis^2 - Do^2\right)\left(\frac{\partial}{\partial t} Ts(x,t)\right) = Cps\,Fs\left(\frac{\partial}{\partial x} Ts(x,t)\right) + \pi\,Do\,Us\,(Tw(x,t) - Ts(x,t)) :$$

Initial and boundary conditions

$$> ibc := Ts(x,0) = 300, Ts(L,t) = 300, Tt(x,0) = 360, Tt(0,t) = 360, Tw(x,0) = 330, D_1(Tw)(0,t) = 0, D_1(Tw)(L,t) = 0 :$$

Solution and Results

```
> sol := pdsolve({pde1, pde2, pde3}, {ibc}, numeric, time = t, range = 0 .. L) :
> p1 := sol:-plot(Tt, x =0.5 L, t = 0 ..5, axes = boxed, color = black, legend = ["Tube-side liquid"]) :
  p2 := sol:-plot(Ts, x = 0.5 L, t = 0 ..5, axes = boxed, color = blue, legend = ["Shell-side liquid"]) :
  p3 := sol:-plot(Tw, x = 0.5 L, t = 0 ..5, axes = boxed, color = red, legend = ["Tube wall"]) :
  plots[display](p1, p2, p3, labels = ["Time (s)", "Temperature (C)"], title
      = "Temperature Halfway Along Heat Exchanger", titlefont = [Helvetica, 18, Bold])
```

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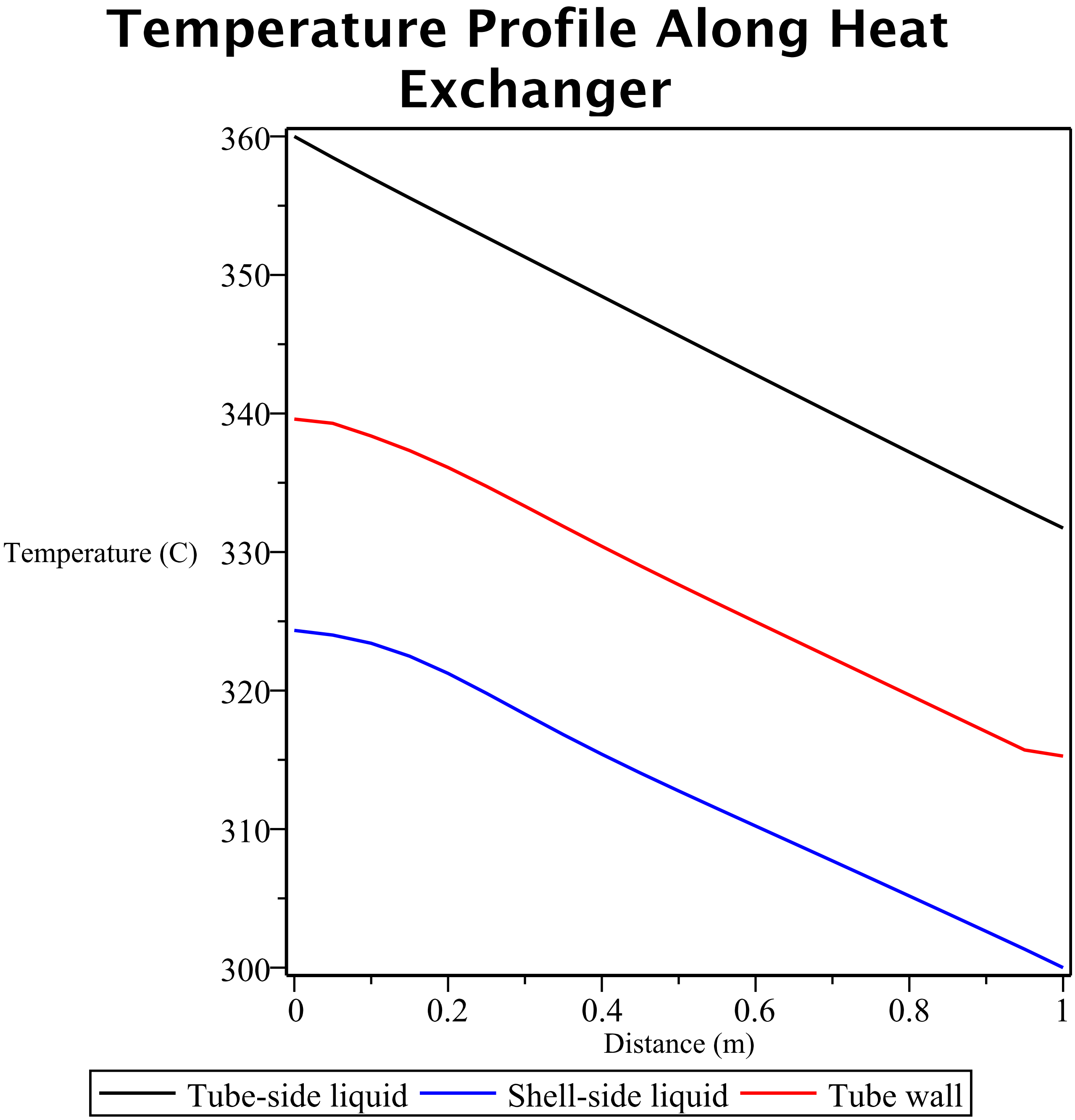
> p1 := sol:-plot(Tt, t = 5, x = 0 ..L, axes = boxed, color = black, legend = ["Tube-side liquid"]) :
  p2 := sol:-plot(Ts, t = 5, x = 0 ..L, axes = boxed, color = blue, legend = ["Shell-side liquid"]) :
  p3 := sol:-plot(Tw, t = 5, x = 0 ..L, axes = boxed, color = red, legend = ["Tube wall"]) :
  plots[display](p1, p2, p3, labels = ["Distance (m)", "Temperature (C)"], title
       = "Temperature Profile Along Heat Exchanger", titlefont = [Helvetica, 18, Bold])
```



Temperature Profile Along Heat Exchanger
Temperature (C)
360
350
340
330
320
310
300
0
0.2
0.4
0.6
0.8
1
Distance (m)
Tube-side liquid
Shell-side liquid
Tube wall